Bounding Mean Throughput Rate and Mean Delay in Office Systems

E. ARTHURS AND B. W. STUCK, MEMBER, IEEE

Abstract-Offices handle a mix of jobs, with each job consisting of **one or more steps. The fundamental ingredients in performance analysis are the job arrival statistics, the service required for each job step, and a scheduling policy, for a given equipment configuration. The approach is hierarchical and can be refined in numerous ways; here we focus on a mean value analysis: the inputs are the mean times required to execute each step of each job. A series of examples illustrate how these ingredients can be used to upper bound the mean throughput rate and lower bound the mean delay associated with each job type.**

I. INTRODUCTION

A I present there is a great interest in improving office
 A productivity (e.g., Mertes [15]). The combination of T present there is a great interest in improving office rising personnel costs (e.g., Engel *et al.* [7]) and falling election of the first is *synthesis*, where goals are stated along with differential personnel costs (e.g., Engel *et al.* [7]). The combination of the first is strong incentives to design more cost effective offices.

An office is an example of a distributed information processing system (cf. Ziegler [21]), exhibiting a variety of tasks that are executed asynchronously and concurrently. These activities are typical of any data communication systems, and comprise data gathering, data manipulation, data communication, data analysis and display, and decision or policy making. On the other hand, office systems are fundamentally *complex,* making it quite important to be systematic in order not to overlook anything. This requires controlled experimentation and measurement, coupled with the formulation of hypotheses or models to explain behavior, as well as analysis.

Here we focus on one technique for bounding the mean throughput rate and mean delay of an abstraction of an office system. This is only one factor among many others, such as cost, flexibility, and reliability which must be considered in choosing one approach over another for a given office (eg., bush [3], Here we focus on one technique for bounding the mean [9], Nutt and Ricci [16]), measuring actual or simulated oper-
throughput rate and mean delay of an abstraction of an office ation, and then repeating this pro Ziegler [21]). We drop these other factors from consideration from this point on in the interest of brevity. We attempt to briefly and nonexhaustively survey *some* of the many aspects of performance analysis.

Broadly speaking, there are three reasons for wanting to quantify performance in an office.

1) In an existing office, it is often possible to modify existing scheduling policies to improve performance at an acceptable cost. An example would be to change from one secretary per department for word processing to a pool of secretaries handling word processing for a set of departments.

Manuscript received March 21, 1981; revised September **1,** 1981. The authors are with Bell Laboratories, Murray **Hill, NJ** 07974.

2) In an office handling a fixed set of job types, different equipment configurations can accomplish these jobs at different costs: which should be chosen? An example would be to compare using electronic mechanical typewriters versus electronic word processors coupled to a shared printer to handle office word processing (cf. Shackil [19]).

3) Comparisons are often desired between current operations and wholly new *modus operandi.* An example would be using hand delivered internal mail versus electronic mail to route internal memoranda and reports (Engel *et al.* [7], Gardner [10]).

1. INTRODUCTION

I. INTRODUCTION

I. INTRODUCTION

Sardner [10]).
 \blacksquare
 \blacksquare The present there is a great interest in improving office
 \blacksquare The combination of the first is *synthesis*, where goals are stated along To quantify these issues, typically two stages are involved: the first is *synthesis,* where goals are stated along with different alternatives for reaching those goals, while the second is *analysis,* where the performance (here the mean throughput rate of finishing jobs and the mean delay for each stage of job execution) is quantified. Goals may be either oriented toward the total system, such as total number of jobs of a given type that are handled during an hour, or toward an individual, such as the mean delay to handle one stage of a job; along with goals such as these there should be some measure of the rate of finishing jobs and the mean delay for each stage of job
execution) is quantified. Goals may be either oriented toward
the total system, such as total number of jobs of a given type
that are handled during an hour, forth.

Analysis often begins by postulating a set of parameters that carry or capture specific operational aspects, drawing inferences based on these parameters (either by mathematical analysis or by discrete event simulation (e.g., Fishman [8], $[9]$, Nutt and Ricci $[16]$), measuring actual or simulated operation, and then repeating this process until it is felt that additional work is no longer warranted.

We demonstrate how to carry out a part of this process via two examples. The methodology is well known in the area of operations research (e.g., Conway, Maxwell, and Miller [2]) and computer digital systems (e.g., Omahen [17], Denning and Buzen **[5]),** but apparently is not nearly as widespread at present in the office automation area as might be hoped (cf. Ellis and Nutt [6], Nutt and Ricci [16]). The examples presented are deliberately elementary, chosen for tractability. Everything of interest can be represented by formulas. Furthermore, this approach is a natural starting point for virtually any study of office performance, and can be refined in a variety of ways, used to check and bound much more complex analyses or simulations, and can be immediately related to measurements in an actual office. Often data are simply not available to describe the arrival statistics and service required for each step of each job such as would be needed in simulation studies; this suggests using a mean value (distribution free) analysis, rather ge from one secretary used to cneck and bound much more complex analyses or
a pool of secretaries simulations, and can be immediately related to measurements
in an actual office. Often data are simply not available to de-
 than more stringent distributional assumptions, and then assessing performance sensitivity by varying the mean value, rather than investing effort in simulation studies. References ARTHURS AND STUCK: BOUNDING MEAN THROUGHPUT RATE AND DELAY

than more stringent distributional assumptions, and then

assessing performance sensitivity by varying the mean value,

rather than investing effort in simulation Examples and STUCK: BOUNDING MEAN THROUGHPUT RATE AND DELAY

than more stringent distributional assumptions, and then

assessing performance sensitivity by varying the mean value,

rather than investing effort in simulatio Formance of a given configuration (cf. Shackil **[19]**). The
approach adopted here is not exhaustive, but it is fundamental.
approach adopted here is not exhaustive, but it is fundamental. approach adopted here is not exhaustive, but it is fundamental. The examples show that **only** two avenues are available for improving office communication performance, reducing the time to handle a given task (i.e., speedup) and handling two or more tasks simultaneously (i.e ., concurrently).

11. THE FIRST OFFICE SYSTEM MODEL

Consider an office system model composed of three classes of entities, *N* managers, *N* secretaries, and *N* word processing stations (Fig. 1). The sole function of the office is document preparation. There are three steps involved in document preparation (Fig. 2). stations (Fig. 1). The sole function of the office is document
preparation. There are three steps involved in document
preparation (Fig. 2).

1) A manager dictates a draft to a secretary. This step has

a mean duration of

1) A manager dictates a draft to a secretary. This step has a mean duration of T_1 min.

2) The secretary enters the draft into a file using a word

3) The manager originating the document edits and proofs the document at a word processor station until the final corrected version is satisfactory. **This** step has a mean duration of T_3 min.

Our problem is to determine an upper bound for λ , the mean throughput rate (measured in documents per minute) **of** document preparation, from start to finish. The first step in the analysis is to construct a state model for the office system behavior. If we imagine observing the office in operation at a given instant of time, say *t,* we would note at most three kinds of activities, one for each of the three steps. Let the 3) The manager originating the document edits and proofs

the document at a word processor station until the final corrected version is satisfactory. This step has a mean duration

of T_3 min.

The mean throughput rate tem, whose components are nonnegative integers. The statement that the office is in state *J* at time *t* then means that at the time of observation, there were *concurrently* in progress j_1 step one, j_2 step two, and j_3 step three activities. We alert the reader that not all values for *J* are possible. We denote by F_1 the set of *J* vectors which are feasible. As an aid to constructing this set F_1 , we form a step resource requirement table (see Table I).

Each column shows the type and quantity of resources required by each step. Since we have a maximum of *N* units of each resource type (managers, secretaries, and word processors), F_1 is the set of three tuples J *F1 zyxwvutsrqponmlkjihgfedcbaZYXWVUTSRQPONMLKJIHGFEDCBA* is the set of three tuples

 $j_k \in \{\text{nonnegative integers}\}$ $k = 1, 2, 3$ **(1** a)

$$
j_1 + j_2 \le N \tag{1b}
$$

$$
j_2 + j_3 \le N \tag{1c}
$$

$$
j_1 + j_3 \le N. \tag{1d}
$$

Now imagine that we monitor the office system for a time interval of *T* min. For each feasible *J* we denote by $\pi(J)$ the *fraction* of the observation time that the office was in state *J.*

Fig. 2. Office model 1: document preparation steps.

TABLE I STEP RESOURCE REQUIREMENT TABLE

	-- Step Type			
ALC: NO Resource				
Type				
Manager				
Secretary			0	
Word Processor	Ω			

We then have

Example. As an aid to consider the positive real number of elements of the numbers
$$
P
$$
 is a maximum of N units.

\n1

by definition. Let us denote by λT the number of document We then have
 $\pi(J) \ge 0$ $J \in F_1$ $\sum_{J \in F_1} \pi(J) = 1$ (2)

by definition. Let us denote by λT the number of document

preparation completions observed in the time interval (0, T). If

T is sufficiently large (so that t *T* is sufficiently large (so that truncation effects at the end of the observation interval are negligible), we may apply Little's formula (the mean number in a system equals the mean rate of jobs flowing through the system multiplied by the mean time per job in the system) (Little [14] , Conway, Maxwell, and Miller [2, pp. 18-19]) to the step executions. More pre-

= (average throughput rate for step **I)**

• (average duration of step I)
$$
I = 1, 2, 3.
$$

$$
\sum_{J \in F_1} j_I \pi(J) = \lambda T_I \qquad I = 1, 2, 3. \tag{4}
$$

Before proceeding with the general analysis, let us consider a special case to gain insight, where $N = 1$. For this case, it is clear that F_1 consists of just four vectors:

$$
F_1 = \{(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}.
$$
 (5)
Our earlier expressions (3), (4) now become

$$
\pi(1,0,0) = \lambda T_1 \quad \pi(0,1,0) = \lambda T_2 \quad \pi(0,0,1) = \lambda T_3. \quad (6)
$$

$$
1 \ge \pi(1, 0, 0) + \pi(0, 1, 0) + \pi(0, 0, 1)
$$

= $\lambda(T_1 + T_2 + T_3)$. (7)

This yields the desired upper bound on λ

$$
\lambda \leqslant \frac{1}{T_1 + T_2 + T_3} \,. \tag{8}
$$

This is obvious on intuitive grounds: when $N = 1$ only one step may be in progress at any one time, there is no concurrency or parallel execution of tasks, and the total number of minutes required for document preparation is $T_1 + T_2 + T_3$ min.

We now examine the general case of arbitrary positive integer valued *N.* Our problem is to maximize the mean throughput rate λ over the feasible $π$

$$
\lambda_{\max} = \max_{\pi(J), J \in F_1} \lambda. \tag{9}
$$

This maximization is subject to the following constraints:

$$
\sum_{J \in F_1} j_I \pi(J) = \lambda T_I \qquad I = 1, 2, 3 \tag{10a}
$$

$$
\sum_{J \in F_1} \pi(J) = 1 \qquad \pi(J) \geq 0. \tag{10b}
$$

A general approach to solving this optimization problem is Going to $N = 2$ *doubles* total system resources, while the to rewrite it as a linear programming problem (Omahen [17], second case results in *three* times the maximum mean through-
Dantzig [4]), and then use one of a variety of standard numer- put rate of the $N = 1$ case, while ical packages for approximating the solution to such problems. maximum mean throughput rate of the $N = 1$ case. The 50 For our simple problem, we shall proceed analytically rather percent gain in maximum mean throughput rate of document

cisely, we assume that characteristics of the solution. First, we observe that (10) MONS ON COMMUNICATIONS, VOL. COM-30, NO. 1, JANUARY 1982
characteristics of the solution. First, we observe that (10)
directly implies (Cairns [3, p. 66]) that a value of λ is possible
if and only if the point $(\lambda T_1, \$ 14

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. COM-30, NO. 1, JANUARY 1982

cisely, we assume that

average number in execution in step I

= (average throughout rate for step I)

= (average throughout rate for step I)

= (est convex set in Euclidean three space containing F_1 , i.e., the *convex hull*, denoted by $C(F_1)$, of F_1 . Since F_1 is a finite we assume that

number in execution in step I

number in execution in step I

verage throughput rate for step I)

verage duration of step I
 $I = 1, 2, 3$.
 $I = 1$ Appendix, we show that $C(F_1)$ is defined by the set points $X =$ More formally, we can write this as (x_1, x_2, x_3) where x_K , $K = 1, 2, 3$ are positive *real* numbers, with

$$
x_K \geqslant 0 \qquad K = 1, 2, 3 \tag{11a}
$$

$$
x_1 + x_2 \le N \tag{11b}
$$

$$
x_2 + x_3 \le N \tag{11c}
$$

$$
x_1 + x_3 \le N \tag{11d}
$$

$$
x_1 + x_2 + x_3 \le \left\lfloor \frac{3N}{2} \right\rfloor \tag{11e}
$$

 $F_1 = \{(0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)\}.$

Our earlier expressions (3), (4) now become
 $\pi(1, 0, 0) = \lambda T_1$ $\pi(0, 1, 0) = \lambda T_2$ $\pi(0, 0, 1) = \lambda T_3$. (6) where [y] denotes the largest integer less than or equal to y,

t the so-called *floor* function. If we substitute $(\lambda T_1, \lambda T_2, \lambda T_3)$ If we add the left- and right-hand sides of (4) we find for (x_1, x_2, x_3) in (11) we immediately get the desired upper bound on maximum mean throughput rate

(7)
\nupper bound on
$$
\lambda
$$

\n(8)
\n
$$
\lambda \leqslant \min \left(\frac{N}{T_1 + T_2}, \frac{N}{T_2 + T_3}, \frac{N}{T_1 + T_3}, \frac{\left\lfloor \frac{3N}{2} \right\rfloor}{T_1 + T_2 + T_3} \right).
$$
\n(12)
\nAs a check, we see that this agrees with (8) with $N = 1$.

As a check, we see that this agrees with (8) with $N = 1$.

As an example how to apply this result, let us assume that $N = 2$ and $T_1 = T_2 = T_3 = 15$ min. We wish to compare the following two configurations.

ond the total number of following two configurations.

nent preparation is $T_1 + T_2 + T_3$ (1) Each manager has his own private secretary and word

processor work station, so there are two independent office

neral case of 1) Each manager has his own private secretary and word processor work station, so there are two independent office systems with $N = 1$.

2) The secretaries and word processor work stations are shared, forming a single pooled office system with $N = 2$.

In the first case, the upper bound on λ will be twice that of a single office:

$$
\lambda_{\text{max,case one}} = \frac{2}{45} \text{ documents per minute.} \tag{13a}
$$

For the second case, using (12) with $N = 2$ we see

$$
λmax,caseone = \frac{2}{45} documents per minute.
$$
 (13a)
For the second case, using (12) with $N = 2$ we see
(10b)
$$
λmax,case two = \frac{3}{45} documents per minute.
$$
 (13b)

put rate of the $N = 1$ case, while the first case is *twice* the than numerically, in order to gain insight into the nature and preparation is entirely due to the policy of pooling (compared

with dedicating) resources. The intuitive idea for the gain is THURS AND STUCK: BOUNDING MEAN THROUGHPUT RATE AND DELAY

with dedicating) resources. The intuitive idea for the gain is

that more work can be done *concurrently*; put differently, in

the first case the *interaction* bet the first case the *interaction* between the available resources was limiting the maximum mean throughput rate, while in the second case these constraints were relatively less severe.

111. THE SECOND OFFICE SYSTEM MODEL

managers, S secretaries, with each secretary having a type-
writer and telephone, and C copiers. There are two types of jobs performed, document preparation (type 1) and telephone call answering (type 2). Document preparation consists of seven steps (Fig. **3): .J@&**

1) *Step (I, I):* and will not generate a new document until the preparation of the previous document is completed. The mean time duration for generating a draft is $T_{1,1}$ min.

2) Step **(1,** *2):* **A** secretary produces a typewritten version of the draft and returns it to the originator. The mean dura-

 $3)$ *Step* $(1, 3)$: The manager corrects the typewritten document. This step has a mean duration of $T_{1,3}$ min and is executed an average of *V* times per document.

4) Step (I, 4): If after Step (1, **3)** changes are required, a secretary makes the changes and returns the document to the originator. This step has a mean duration of $T_{1,4}$ min.

a secretary walks to a copier. The mean time required is $T_{1,5}$ min

6) Step (1, 6): A secretary reproduces the requisite number of copies. The mean duration of time is $T_{1,6}$ min.

7) A secretary returns the document with copies to the originator. This requires a mean time interval of $T_{1,7}$ min.

The telephone call answering job consists of one step:

I) Step (2, **1): A** secretary answers a telephone, talks, and takes any messages. The mean duration of this job is $T_{2,1}$ min.

In this model, we make the natural assumption that the number of managers is greater than or equal to the number of secretaries, and the number of secretaries is greater than or equal to the number of copiers. More formally, we can write

$$
M \ge S \ge C. \tag{14}
$$

We next construct the step requirements table for this office (see Table 11).

In this model, there is one document per manager or a total of *M* documents circulating through the office system, with each document either waiting for one or more resources to become available, or being executed in Steps **(1,** 1) through (1, 7). It is therefore convenient to append an additional step, $(1, 8)$, to our model: Step $(1, 8)$ is the waiting state of a document, and $T_{1,8}$ denotes the mean time a document spends waiting for resources. If we denote the mean throughput rate for document preparation, job type 1, by λ_1 jobs/min, and the mean telephone call answering rate for type 2 jobs by λ_2 jobs/ min, then our goal is to determine upper bounds on λ_1 , λ_2 and lower bounds on $T_{1,8}$. The state of the system at any ϵ instant of time is represented by a nine tuple or vector de- $(16c)$

Fig. 3. Office model 2: document preparation steps.

TABLE I1 STEP RESOURCE REQUIREMENTS

Resource	Step Type								
Type	(1.1)	(1,2)	(1,3)	1,4)	(1,5)	(1,6)	(1,7)	(2,1)	
Manager		0		0	0	0	0	0	
Secretary	0		0						
Typewriter	0		0		0	0	0	0	
Telephone	0	0	0	0	0	0	0		
Copier	٠ $\bf{0}$	0	0	0	0		0	0	

noted by *J,* whose components are nonnegative integers:

$$
J = (j_{1,1}, j_{1,2}, \cdots, j_{1,8}, j_{2,1})
$$

 $j_{I,K} \in \{\text{nonnegative integers}\}, \quad I=1,2; K=1, ..., 8.$ (15)

From the step requirements table and (14) we can write that the feasible set of *J* is denoted by F_2 , while (15) implies the components of $J \in F_2$ are nonnegative integers such that

$$
j_{1,6} \leq C \tag{16a}
$$

$$
j_{1,2} + j_{1,4} + j_{1,5} + j_{1,6} + j_{1,7} + j_{2,1} \leq S \tag{16b}
$$

$$
j_{1,1} + j_{1,2} + j_{1,3} + j_{1,4} + j_{1,5} + j_{1,6} + j_{1,7} + j_{1,8} = M.
$$
\n(16c)

In the Appendix, we show that $C(F_2)$, the convex hull of F_2 , is given by the set of nine tuples with *real* valued nonnega-**F2, is given by the set of nine tuples with** *z***ee** *FRANSACTIONS* **ON COMMUNICATIONS, VOL. COM-30, NO. 1, JANUARY 1982

In the Appendix, we show that** $C(F_2)$ **, the convex hull of
** F_2 **, is given by the set of nine tuples w** The Appendix, we show that $C(F_2)$, the convex hull of F_2 , is given by the set of nine tuples with *real* valued nonnegative entries $X = (x_{1,1}, \dots, x_{1,8}, x_{2,1})$ that satisfy the follow-ing constraints. In the Appendix, we show that $C(F_2)$, the convex hull of
 F_2 , is given by the set of nine tuples with *real* valued nonnegative entries $X = (x_{1,1}, ..., x_{1,8}, x_{2,1})$ that satisfy the follow-

ing constraints.

$$
x_{1,1}, x_{1,2}, \cdots, x_{1,8}, x_{2,1} \ge 0 \tag{17a}
$$

$$
x_{1,6} \leq C \tag{17b}
$$

$$
x_{1,2} + x_{1,4} + x_{1,5} + x_{1,6} + x_{1,7} + x_{2,1} \le S \tag{17c}
$$

$$
x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{1,6} + x_{1,7} + x_{1,8} = M.
$$

$$
(17d)
$$

Assuming that Little's formula holds, we can write **Fig. 4.** Office **model 2:** telephone call **answering** steps.

$$
\sum_{J \in F_2} j_{1,K} \pi(J) = \lambda_1 T_{1,K} \qquad K = 1, 2, 5, 6, 7, 8 \qquad (18a)
$$

$$
\sum_{J \in F_2} j_{1,3} \pi(J) = \lambda_1 V T_{1,3} \tag{18b}
$$

$$
\sum_{V \in \mathcal{F}_2} j_{1,4} \pi(J) = \lambda_1 (V - 1) T_{1,4} \tag{18c}
$$

$$
\sum_{J \in F_2} i_{2,1} \pi(J) = \lambda_2 T_{2,1}.
$$
 (18d)

Equations (18b) and (18c), which are associated with Steps (1, 3) and (1, 4), respectively, reflect the fact that there are an mean number of $V, V \ge 1$ steps of type (1, 3) per job 1 $T_{\text{doc}} \ge \frac{T_{\text{doc}}}{\lambda_{1,\text{max}}}$ (23)
 $J \in F_2$ $i_{2,1} \pi(J) = \lambda_2 T_{2,1}$. (18d)

Equations (18b) and (18c), which are associated with Steps

(1, 3) and (1, 4), respectively, reflect the fact that there are we can use (20a) t $\sum_{J \in F_2} i_{2,1} \pi(J) = \lambda_2 T_{2,1}.$ (18d)

Equations (18b) and (18c), which are associated with Steps convex polygon (Fig. 4). For a fixed value of $\lambda_2 (\lambda_2 < S/T_{2,1})$,

3) and (1, 4), respectively, reflect the fact that the

and $(V-1)$ steps of type $(1, 4)$ per job 1.
The values λ_1 , λ_2 , $T_{1.8}$ are possible if and only if the point

$$
(\lambda_1 T_{1,1}, \lambda_1 T_{1,2}, \lambda_1 VT_{1,3}, \lambda_1 (V-1)T_{1,4}, \lambda_1 T_{1,5}, \lambda_1 T_{1,6}, \lambda_1 T_{1,7}, \lambda_1 T_{1,8}, \lambda_2 T_{2,1})
$$
\n(19)

is a member of the convex hull of the feasible set F_2 . Substituting into (17) we have

$$
\lambda_1 \le \lambda_{1,\max} = \min\left(\frac{M}{T_M}, \frac{S - \lambda_2 T_{2,1}}{T_S}, \frac{C}{T_{1,6}}\right) \tag{20a}
$$

$$
\lambda_2 \le \lambda_{2,\text{max}} = \frac{S}{T_{2,1}}\tag{20b}
$$

$$
T_{1,8} \ge \frac{M}{\lambda_{1,\text{max}}} - T_M \tag{20c}
$$

where T_M is given by

$$
T_M = T_{1,1} + T_{1,2} + VT_{1,3} + (V - 1)T_{1,4} + T_{1,5}
$$

+ $T_{1,6} + T_{1,7}$ (21a)

and T_S is given by

$$
T_S = T_{1,2} + (V - 1)T_{1,4} + T_{1,5} + T_{1,6} + T_{1,7}.
$$
\n(21b)

The mean delay in document preparation T_{doc} is the interval

of time measured from the start of document generation to the delivery of the hard copies is given by

$$
T_{\rm doc} \ge T_M + T_{1,8}.\tag{22}
$$

Using (20c) we see

$$
T_{\text{doc}} \ge \frac{M}{\lambda_{1,\text{max}}} \tag{23}
$$

We remark that the set of feasible points (λ_1, λ_2) form a convex polygon (Fig. 4). For a fixed value of $\lambda_2(\lambda_2 \leq S/T_{2,1})$, we can **use** (20a) to determine the potential bottlenecks.

1) Managers are the bottleneck:

limit

\n
$$
\lambda_{1,\max} = \frac{M}{T_M} \tag{24a}
$$
\n(19)

\n2) Secretaries are the bottleneck:

\n
$$
\sum_{n=1}^{\infty} \lambda_n = \frac{M}{T_M} \tag{24b}
$$

2) Secretaries are the bottleneck:

$$
\lambda_{1,\max} = \frac{S - \lambda_2 T_{2,1}}{T_S} \,. \tag{24b}
$$

3) Copiers are the bottleneck:

$$
\lambda_{1,\max} = \frac{C}{T_{1,6}} \tag{24c}
$$

We illustrate the bounds on λ_1 and T_{doc} as a function of the number of secretaries S

$$
S > \max(\lambda_2 T_{2,1}, C) \tag{25}
$$

in Figs. **5** and *6.* The feasible operating regions are also shown.

This example, while considerably more complex than the first example, shows the importance of being systematic in enumerating all possible states because nothing will be overlooked. Furthermore, we have shown how to extend the first example to handle multiple job types and multiple visits to each step of a given job type.

IV. CONCLUSIONS

A performance study of an office system may be carried out in at least one of three ways:

Fig. 5. Office model 2: feasible region **of mean throughput rates.**

Fig. 6. Office model 2: mean throughput and mean delay feasible regions.

1) mean value analysis as described here (e.g., Omahen [**171** , Denning and Buzen **[5]**),

2) Jackson queueing network analysis (eg., Kelly [**13]),**

3) discrete event simulation model (e.g., Fishman [8], $[9]$, **Nutt** and Ricci $[16]$.

In this paper we have demonstrated the ability of the mean value analysis to present a clear picture of the dependence of office system performance on the values of the model parameters. The mean value analysis is a simple, flexible, inexpensive approach to performance analysis and should always be used, even if it is required to supplement the analysis with one or both of the other techniques. The other approaches quantify the impact of fluctuations about mean values on performance, refining thg mean value analysis.

EXAMPLE 12 the impact of fluctuations about mean values on performance,
RATE THE EXAMPLE 12 in a vacuum: whichever approach or combination of
RATE in a vacuum: whichever approach or combination of
RATE data gathere The utility or validity of any of these approaches cannot be judged in a vacuum: whichever approach or combination of methods is most appropriate must be judged in terms of the data gathered and the measurements, and how the data are used to draw inferences concerning cause and effect phenomena, coupled with the spectrum of practical feasible alternatives. The mean value approach presented here is simply one tool for carrying out this complex decision making process.

APPENDIX

We briefly sketch the proofs that **(1 1)** and **(17)** define the convex hull of the feasible sets for their respective models, denoted here by model one and model two, respectively. Equations **(1 1)** and **(17)** each defme a convex polyhedron which we shall call G_1 for model one and G_2 for model two.
It is clear that G_K , $K = 1, 2$ contains F_K , $K = 1, 2$ so by **APPENDIX**
We briefly sketch the proofs that (11) and (17) define the
convex hull of the feasible sets for their respective models,
denoted here by model one and model two, respectively.
Equations (11) and (17) each defin definition *zyrkuptiffer the proofs that (11) and (17) define the ll of the feasible sets for their respective models, ere by model one and model two, respectively.
(11) and (17) each define a convex polyhedron shall call* G_1 *for*

$$
C(F_K) \subset G_K \qquad K = 1, 2. \tag{A1}
$$

The vertex set of G_K , $K = 1, 2$ is a subset of the points of G_K , $K = 1$, 2 which satisfies three of the inequalities in (11) [respectively, **(17)]** with equality. It is easy to verify that all such points belonging to G_K , $K = 1, 2$ must have integral coordinates and therefore belong to F_K , $K = 1, 2$. Hence,

$$
C(F_K) \equiv G_K \qquad K = 1, 2. \qquad \text{Q.E.D.}
$$

(A21

REFERENCES

- **[I] V. Bush, "As we may think,"** *Arlunric Monthly,* **vol. 176, pp. 101-108, July 1945.**
- **[2] R. W. Conway, W. L. Maxwell, and L. W. Miller, "Little's formulary in a** *Theory of Scheduling.**Theory of Scheduling.* **in** *Theory of Scheduling.* **Reading, MA: Addison-**
 We Scheduling. Property *Scheduling*. **Reading, MA: Addison-**
 Wesley, 1967, pp. 18–19.
 S. S. Ca Wesley, 1967, pp. 18-19.
- **[3]** *S. S.* **Cairns,** *Inrroducrory Topology.* **New York: Ronald, 1968.**
- **[4]** *G.* **B. Dantzig,** *Linear Programming and Extensions.* **Princeton, NJ: Princeton Univ. Press, 1963.**
- **[5] P. J. Denning and J. P. Buzen, "The operational analysis of queueing network models,"** *Comput. Surveys,* **vol.** 10, **pp. 225- 261, 1978.**
- **[6] C. A. Ellis and G. J. Nutt, "Office information systems and computer science," Compur.** *Surveys,* **vol. 12, pp. 27-60, 1980.**
- **[7] G. H. Engel, J. Groppuso, R. A. Lowenstein, and W. G. Traub, "An office communications system,"** *IEM Syst. J.,* **vol. 18, no. 4 pp. 402-431, 1979.**
- **[8] G.** *S.* **Fishman,** *Concepts and Merhods in Discrete Evenr Digital Simularion.* **New York: Wiley, 1973.**
- -, Principles of Discrete Event Simulation. New York: Wiley, $[9]$ 1978.
- $[10]$ P. C. Gardner, "A system for the automated office environment,"
IBM Syst. J., vol. 20, pp. 321–345, 1981.
- P. Hayes, **E.** Ball, and R. Reddy, "Breaking the man-machine
- G. A. Helander, "Improving system usability for business pro- $[12]$ fessionals," *IBM Syst. J.*, vol. 20, pp. 294-305, 1981.
- $[13]$ **F.** P. Kelly, *Reversibility and Stochastic Networks.* New York: Wiley, 1979.
[14] J. D. C. Little, "A proof of the queueing formula $L = \lambda W$," Oper.
- *Res.,* vol. 9, pp. 383-387, 1961.
- L. H. Mertes, "Doing **your** office over electronically," *Harvard Bus. Rev.,* vol. 59, pp. 127-135, 1981.
- G. J. Nutt and **P.** A. Ricci, "Quinault: An office modeling system," *Computer,* vol. 14, pp. 41-58, 1981.
- **K.** Omahen, "Capacity bounds for multiresource queues," *J. Ass. Compur. Mach.,* **vol.** 24, pp. 646-663, 1977.
- ed. Bedford, MA: Digital, 1979. M. Phister, Jr., *Data Processing: Technology and Economics,* 2nd
- **A. F.** Shackil, "Design case history: Wang's word processor," *IEEE Spectrum,* vol. **18,** pp. 29-33, Aug. 1981.
- R. P. Uhlig, D. J. Farber, and **J.** H. Bair, *The Office* **of** *the Future:*

Communication and Computers. Amsterdam, The Netherlands: North-Holland, 1979.

THERE TRANSACTIONS ON COMMUNICATIONS, VOL. COM-30, NO. 1, JANUARY 1982
 zommunication and *Computers*. Amsterdam, The Netherlands:

1978.

P. C. Gardner, "A system for the automated office environment," [21] K. Ziegler, *IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. COM-30, NO. 1, JANOART 1982*
 I978.
 I978.
 P. C. Gardner, "A system for the automated office environment," [21] *K. Ziegler, Jr., "A distributed information system study,"* **Examples of Discrete Event Simulation.** New York: Wiley,

1978.
 P. C. Gardner, "A system for the automated office environment," [21] **K.** Ziegler, Jr., "A distributed information system study," *IBM*
 IBM Syst. J., 1211 **K.** Ziegler, **Jr.,** "A distributed information system study," *IBM* **Syst.** *J.,* vol. **18,** pp. 374-401, 1979.

*

E. Arthurs received the Ph.D. degree in electrical engineering fromrhe Massachusetts Institute of Technology, Cambridge, in 1957.

J. C. Gardner, "A system for the automated office environment," [21] K. Ziegler, Jr., "A distributed information system study," IBM Syst. J., vol. 18, pp. 374-401, 1979.
 D. Hayes, E. Ball, and R. Reddy, "Breaking the He was a faculty member of the M.I.T. Department **of** Electrical Engineering from 1957 until joining Bell Laboratories, Murray Hill, NJ, in 1962. He has worked on a variety **of** communication and computer systems. \bigstar

B. W. Stuck (S'67-M'72) received the S.B.E.E. and S.M.E.E. degrees in 1969 and the Sc.D. degree in 1972, all from the Massachusetts Institute **of** Technology, Cambridge.

Since joining Bell Laboratories in 1972, he has worked on a variety of problems associated with digital communications and computer and communication systems.