TABLE I
A SHORT TABLE OF R_i

i	R_i	i	R_i
0	1	16	$-\frac{1}{3}$
1	$-\frac{1}{3}$	17	$\frac{1}{12}$
2	$-\frac{1}{3}$	18	$\frac{1}{6}$
3	$\frac{1}{3}$	19	$-\frac{1}{12}$
4	$-\frac{1}{3}$	20	0
5	0	21	$\frac{1}{12}$
6	$\frac{1}{3}$	22	$-\frac{1}{6}$
7	0	23	$-\frac{1}{12}$
8	$-\frac{1}{3}$	24	$\frac{1}{3}$
9	$\frac{1}{6}$	25	$-\frac{1}{12}$
10	0	26	$-\frac{1}{6}$
11	$-\frac{1}{6}$	27	$\frac{1}{12}$
12	$\frac{1}{3}$	28	0
13	$-\frac{1}{6}$	29	$-\frac{1}{12}$
14	0	30	$\frac{1}{6}$
15	$\frac{1}{6}$	31	$\frac{1}{12}$

Because each series in (27) is periodic, each series is wide-sense stationary and thus, if we consider the TMS started at a random value x, $x \geqslant 0$, the sequence

$$\{\sigma(i)\}_{i=x}^{\infty} \tag{28}$$

will also exhibit wide-sense stationarity. Thus, we have a random process that exhibits wide-sense stationarity and displays the power spectral density and autocorrelation properties of the TMS.

ACKNOWLEDGMENT

We thank one of the reviewers for suggesting a shorter proof for (20), which we have incorporated in this paper.

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A Modified Access Policy for ETHERNET™ Version 1.0 Data Link Layer

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Abstract—**A modification to the access policy for ETHERNET™ Version 1.0 is proposed. By placing additional restrictions (compared with ETHERNET) on each station that is attempting to transmit, this access policy makes it more likely a successful message transmission will occur, and in less time, than the published Version 1.0. Theoretical analysis substantiates these claims.**

I. INTRODUCTION

In [9] (see also [2], [3], [10], [15]) a particular policy is described for resolving contention for a shared data using serial data transmission. Roughly speaking, this policy operates as follows.

- All stations (both those with messages to transmit and those without messages) sense the state of transmission medium; if energy is detected, a station that is ready to transmit a message will wait until the transmission medium becomes idle before attempting to transmit.
- If a station senses the transmission medium idle and has a message to send, it attempts to transmit the message.
- If two or more stations attempt to seize the transmission medium within a given time interval called the *collision window*, their transmission attempts will interfere with one another, and a *collision* between these message transmission attempts is said to occur. Transmission ceases, and each station will compute a time interval for reattempting to transmit that message.

A detailed description of this policy is given elsewhere for those interested [9, sec. 6].

In [9], the retransmission times that each station computes involve generating a random variable from a discrete uniform distribution with support on the integers $\{0, \cdots, 2^{\min(10, N_{\text{collision}})}\}$ for $N_{\text{collision}} \leqslant 16$ where $N_{\text{collision}}$ is the number of collisions that a station undergoes in attempting to transmit a given message. Once the random number is generated, the retry time interval equals the random number multiplied by the *slot time* denoted by T_{slot}. The slot time is chosen to account for a worst case message propagation from one end of the bus to the other, plus cabling and electronic circuitry transients. For $N_{\text{collision}} > 16$, the mean retry time interval is infinite. This policy attempts to spread out attempts in time, in order to ensure that eventually a message will be successfully transmitted, at the cost of increasing message delay. The rationale behind this policy is sketched elsewhere [2]. A simulation study that suggests how to choose the different parameters with this access method is recommended reading [1].

Our contribution is to modify this access policy. By imposing additional restrictions byond [9] on stations being allowed to make transmission attempts, we make it *more* likely a successful transmission will take place *sooner*. This is based on extant ideas in the literature (e.g., [5]-[7], [11], [12]). Furthermore, we *quantify* via theoretical analysis its performance. Typical theoretical results are as follows: if three message transmission attempts collide, the mean-time delay until the start of the first sucessful transmission is 3.6 percent smaller here than in [2]; if ten message transmission attempts collide, the mean time delay until the start of the first successful transmission is 14.5 percent smaller here than in [2]. We stress that no experimental evidence is available to confirm these findings, only theoretical analysis. Our intent is to encourage others to investigate this with controlled experimentation on actual systems.

II. DESCRIPTION OF POLICY

Each station can sense locally if the medium is *idle* or *busy*. The transmission medium can be busy with an initial transmission attempt, with collision resolution (measured either from the initial collision after leaving idle, or after a successful transmission until the end of a successful transmission),

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or with a successful transmission. To implement this is a straightforward modification to [9] (see also [3], [15], and is omitted in the interest of brevity.

A. Proposed Access Method

The proposed access method operates as follows.

- If the medium is busy when a station is ready to attempt to transmit, that station will defer all attempts until a successful transmission occurs.
- Once a successful transmission occurs, all stations will sense the bus for carrier or electromagnetic energy; if there is no carrier, all stations ready with a message will attempt to transmit.
- If a station is successful, the process begins anew.
- If a station is not successful, all stations involved in the collision compute a retry time interval using the published method [9].

B. Comments

Once a collision occurs, only those stations involved in it can participate; new attempts to seize the transmission medium are deferred until a message is successfully transmitted. In particular, a station involved in the initial collision that defers to new transmission attempts will defer until the collision resolution ends with a successful transmission.

If one station successfully seizes the transmission medium, there are no differences in operation between [9] and the proposed access method. If two or more stations attempt to seize the transmission medium within a collision window time interval, each station involved in the collision will compute a retry time interval according to the prescribed retry policy [9]. Each station will listen to the transmission medium while it is waiting for its retry time interval to end, looking for one of three conditions:

- A successful transmission occurs, so each station resets its retry time interval to 0 and waits until the end of the successful transmission before attempting to seize the transmission medium.
- Two or more stations attempt to transmit and their attempts collide; all stations involved in the collision will compute a retry time interval and repeat the process; all stations not involved in the collision will wait until one successful transmission occurs.
- The transmission medium remains idle and the retry time interval ends.

C. Failure Mode

The proposed access method has a failure mode that [9] does not: if the collision resolution process begins, but all of the stations involved fail to ever successfully transmit a message (due to a variety of reasons, such as a momentary power failure, transient hardware error, and so forth), then no station is allowed to transmit. To overcome this, each station also has a *timer*. At the start of collision resolution, this timer is started by every station; if no message is successfully transmitted when the timer expires, all stations that are ready with a message will start the collision resolution process over again. For example, the timer may be set equal to the worst case collision resolution time interval: 16 collisions, each occurring at the largest possible retry time plus one successful message transmission of a maximum size frame.

$$T_{\text{timeout}} = \sum_{K=1}^{16} \min\,[K,\ 10]\, T_{\text{slot}} + T_{\text{max frame}}. \tag{1}$$

Finally, initialization of all counters is required at power up, and for testing purposes. This can be handled by straightforward modifications to [9] and is omitted in the interest of brevity.

III. ANALYSIS

Assume that at a given point in time, N stations attempt to initially seize the transmission medium because each sensed it idle, or each sensed completion of a successful transmission, or each timed out. Assume that all N attempts collide with one another. Each of the associated stations will compute a retry time interval denoted by t_J for station $J = 1, \cdots, N$. Without loss of generality, we order these such that $t_I < t_J, I < J$. At time t_1, one station will attempt once again to seize the transmission medium; this station will succeed if and only if no other station attempts to do likewise within the duration of a collision window, denoted by T_{slot}.

For ease of analysis, we assume each station involved in the initial collision computes its retry time interval from a common exponential distribution with mean $1/E(T_{\text{retry}})$. This is felt to be a reasonable approximation when large numbers of stations are attempting to transmit (which is precisely the region of interest in this analysis).[1]

The probability that one or more stations, denoted by the random variable N', will attempt to seize the transmission medium in a time interval of duration T_{slot} following t_1, is given by

$$\text{Prob}\,[N' = K] = \binom{N-1}{K-1} \cdot [1 - \exp(-T_{\text{slot}}/E(T_{\text{retry}}))]^{K-1} \cdot \exp\,[-(N-K)T_{\text{slot}}/E(T_{\text{retry}})] \qquad K > 0. \tag{2}$$

The random variable N' is distributed according to a binomial distribution with mean

$$E[N' \,|\, N] = (N-1)[1 - \exp(-T_{\text{slot}}/E(T_{\text{retry}}))] + 1. \tag{3}$$

For example, if we choose to fix the mean retry time such that

$$1 - \exp\,[-T_{\text{slot}}/E(T_{\text{retry}})] = \tfrac{1}{2} \tag{4}$$

then we see that

$$E[N' \,|\, N] = \frac{N+1}{2} < N. \tag{5}$$

In this sense, at every collision epoch we will have decreased the mean number of competing stations by one-half, and, hence, thinned the number of stations attempting to transmit. This will increase the mean throughput rate as the offered load increases faster than the published access method.

A different way of quantifying the improvement is to note that the number of time slots wasted in collision resolution until a successful message transmission is proportional to the *logarithm* of the number of messages in the initial collision. On the other hand, all published analyses (e.g., [4]) show that the number of time slots wasted in collision resolution is at least *linear* in the number of messages in the initial collision.

We can also parallel the original mathemtical analysis of the maximum mean throughput rate for the published access method [2] as follows. We assume the stations connected to the transmission medium are synchronized by a master clock, with the basic clock period being T_{slot}. Transmission attempts

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[1] This is the so-called generalized central theorem of probability theory: the distribution of a superposition of events generated by independent identically distributed sources converges to an exponential distribution.

can only begin at the start of a time slot, and each station will know whether or not the attempt was successful in seizing the transmission medium at the end of the time slot. If the attempt was successful, the station will hold the channel for as many time slots as are required to finish message transmission. If the attempt was unsuccessful, the stations involved will retry according to the modified policy proposed here. The stations involved in the initial collision will retry in the next time slot with probability Q_K where K is the number of stations involved in the collision, and will repeat this retry procedure after delaying until the next time slot with probability $1 - Q_K$.

We denote by W_K, $K = 1, 2, \cdots$ the mean waiting time (not including successful message transmission) from the start of an initial collision until the first successful message transmission, given that K messages initially collide. W_K, $K = 1, 2, \cdots$ is measured in time slots. Granted this, $W_1 = 0$ while for $K > 1$

$$W_K = \sum_{J=0}^{\infty} \sum_{K=1}^{N} (1 - Q_N)^{NJ} \binom{N}{K} (Q_N)^K (1 - Q_N)^{N-K} \cdot [1 + J + W_K]. \tag{6}$$

In contrast, the original published mathematical analysis [2] showed that $W_1 = 0$ while

$$W_N = \sum_{J=0}^{\infty} [1 - (1 - Q_N)^{N-1}]^J (1 - Q_N)^{N-1} [J + 1]. \tag{7}$$

It is possible to minimize W_N by choosing Q_N appropriately. In [2], it was shown that Q_K can be approximated by $1/K$ as $K \to \infty$, which naturally suggested an adaptive retry policy as described in [9]. The same type of adaptation to large number of transmission attempts, but with a different implementation, in principle occurs here. In the interest of brevity, we fix $Q_K = 1/K$ for *all* values of K for the access method in [9] and that proposed here. For the access method proposed here plus the modified retry policy, we choose $Q_K = 1/2$.

For [2], we see

$$\lim_{N \to \infty} W_N = \left(1 - \frac{1}{N}\right)^{-(N-1)} = e = 2.718281828 \cdots. \tag{8}$$

For example, in [2], $W_2 = 2.00$ and $W_3 = 2.25$. In contrast, for the modified policy proposed here, we see $W_2 = 2.00$ and $W_3 = 2.17$.

Fig. 1 plots the mean waiting time to the start of the first successful transmission after a collision, versus the number of messages involved in the initial collision. As is evident, the modified access method offers a shorter mean time to a successful transmission compared with the published version using the current retry policy.

IV. CLOSING COMMENTS

The retry policy proposed here is analyzed assuming Q_K is known; a similar assumption is made in [2]. In either case, this is *impossible*, in the sense that the system is intrinsically distributed, so this information is not available to the stations, and must be *estimated*. In [9] this statistic is computed by choosing a random number from a distribution with mean proportional to the number of collisions per message. It is well known that this policy (as well as the one proposed here) is *unstable* in the sense that the fraction of time a message is delayed less than any finite threshold becomes arbitrarily close to one as the *number* of stations involved in the collision resolution becomes arbitarily large [13]-[16]. What is perhaps more surprising is that there exist realizable policies (e.g., [6]-[8], [11], [12]) for which this is *not* true, and we call these policies *stable* in this sense.

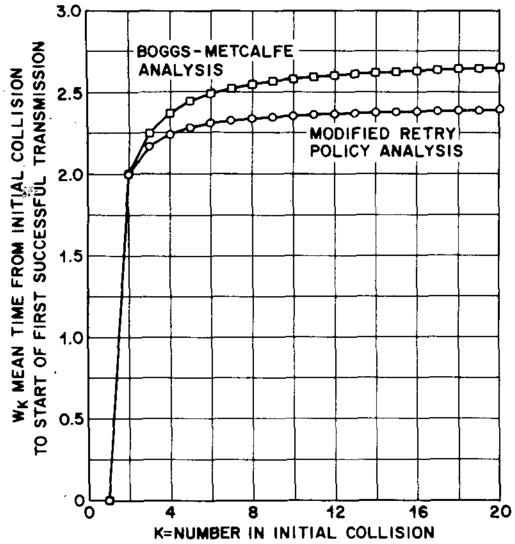

Fig. 1. Mean time to start successful transmission versus number of messages in initial collision.

The reader is cautioned that the transmission medium access policy is only one factor among many that must be considered in the design of systems that allow diverse devices to communicate among one another [14].

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